Indian J Phys. 80 (9), 947 (2006)

PHYSICAL REVIEW VOLUME 106, NUMBER 1 APRIL 1, 1957

Singular State in Relativistic Cosmology

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(Received February 5, 1957)

As is well known, the isotropic cosmological solutions of general relativity start from a singular state in the finite past. In a recent paper Komar[1] has investigated the question as to whether this singularity persists under more general circumstances and has found that such a singularity does occur unless one

(i) introduces a cosmological term or a negative pressure term or (ii) considers spaces which do not have a set of geodesically parallel space-like hypersurfaces for all times. [These are conditions (B) and (C) as given in the conclusion of Komar's paper.]

It is interesting to point out that this result was explicitly given by the present author[2] in a previous paper of which Komar is apparently unaware. There it was shown that under very general circumstances, the expansion is controlled by the following equation[3]:

$$(1/G)(\partial^2 G/\partial t^2) = (\Lambda - 4\pi\rho - \phi^2 + 2\omega^2)/3, \qquad (1)$$

where $G^6 = -\det g_{ik}$, $\phi^2 \geqslant 0$ (the equality sign occurring in the case of isotropic expansion), and ω is the local angular velocity. In the case $\omega = 0$ [which corresponds to the existence of a set of geodesically parallel space-like hypersurfaces[4]; see condition (B) of Komar], and $\Lambda = 0$ [see condition (C) of Komar], we get

$$(1/G)(\partial^2 G/\partial t^2) = (-4\pi\rho - \varphi^2)/3, \qquad (2)$$

so that with $\rho \geqslant 0$, G cannot have any minimum and one has to start from the singular state $G = 0$ at a finite time in the past (see the last paragraph of Sec. III of reference 2).

[1] A. Komar, Phys. Rev. 104, 544 (1956)
[2] A. Raychaudhuri, Phys. Rev. 98, 1123 (1955).
[3] In the deduction the stress components were assumed to vanish; however, the introduction of these would mean only the replacement of ρ by $T_{44} - T^1{}_1 - T^2{}_2 - T^3{}_3$ in Eq (1) As $T_{44} - T^1{}_1 - T^2{}_2 - T^3{}_3 \geq 0$, the conclusion remains unaffected
[4] K. Gödel, Revs. Modern Phys. 21, 447 (1949)